

REMARKS

The following paragraph numbers correspond to the same paragraph numbers in the subject Detailed Office Action. Applicant appreciates the useful consideration given to the present application by the examiner.

1. Claim Rejections - 35 USC § 101

Claims 1,2,3 have been amended to read as method claims for implementation which are directed toward statutory subject matter which is the implementation of transmitter and receivers for cellular communications links.

2. Claim rejections

In claims 1,2,3 the amended claims start by replacing "A means for the design of" with "A method for implementation of" and the remainder of the claims are amended to provide "methods for implementation". It is believed these amended claims fall into the statutory categories of a method, process, system.

3. Claim rejections

It is believed that the amended claims 1,2,3 provide adequate punctuation.

4. Drawings

Drawings have been amended to correct my mistakes. FIG. 1-2 are designated as "Prior Art" in the legend. FIG. 3 is a new figure depicting the orthogonal Wavelet multiple access OWDMA frequency spectrum which has the same channel spacing and band filter as FIG. 1 for orthogonal frequency division multiple access OFDMA but with channelization filters as depicted in the figure versus the un-filtered harmonic tones for OFDMA. Figures are marked as "Deleted Sheet" when corrected and the corrected

figures are marked as "Replacement Sheet" for the amended figures. FIG. 3 has a spelling correction in the label 16. FIG. 5 has a replacement with the data plots distinguishable with no color. FIG. 7 has a title change in the replacement FIG. 7A and a "New Sheet" is added as FIG. 7B to incorporate additional transmit mapping information available in the prior application 10/266,257 filed 10/08/2002.

5. Abstract

The abstract has been amended to be narrative in form to describe the disclosure sufficiently to assist readers in deciding whether or there is a need for consulting the full text for details, using clear and concise language with no repetition of the information in the title.

6. Abstract

The abstract has been amended to be 126 words and the phrase "means for" has been deleted.

7. Specification

The applicant requests withdrawal of the rejection in view of the response to correct mistakes in the marked up substitute specification, as documented in the following.

8. Specification

The word "shaded" in reference to Figures 5,6,8 or 9 has been removed from the marked up substitute specification in page 21 line 30, in page 40 line 18, and in page 45 lines 3 and 5.

9. Specification

Acronyms are clearly defined the first time in the marked up substitute specification where they are used.

"ACI" in page 23 line 16 is the "adjacent channel

interference",  
"ISI" in page 24 line 2 is the "Inter-symbol  
Interference",  
"FWT" in page 28 lines 15 and 16 is the "fast multi-  
resolution complex Wavelet transform",  
"FCT" in page 35 line 25 is the "fast code transform",  
"CRC" in page 40 lines 23-24 is a "cyclic redundant code",  
"Ampl." In page 40 line 26 is the "amplitude",  
"Rx" in page 45 line 7 is "received",

#### 10. Specification

Misspelled words are corrected in the marked up substitute specification.

"channelizaation" in page 9 line 30 is corrected to  
"channelization",  
"imbalances" in page 12 line 8 is corrected to the singular  
word "imbalance",  
"equations" in page 22 line 11 is deleted since it is  
redundant.  
"multiply" in page 39 line 8 is correct and should not be  
changed to "multiple" since it is part of the text  
"the real multiply complexity metric" which describes  
a metric that measures the number of algebraic  
"multiply" operations,  
"implementes" in page 45 line 29 is incorrect and is  
deleted along with the accompanying text.

#### 11. Specification

The following is an explanation of amendments made to the marked up substitute specification. No new material has been added in any of the following amendments.

<u>Amendment</u>	<u>Corrections</u>
Page 1 lines 1-13,20,27,31-43	Format, spacing, continuation Statement
Page 2 lines 1-8,21-24,27-28	Text, spelling, spacing
Page 4 lines 5,7-9,14,16	Spacing
Page 5 lines 27,29,31-32	Spacing, text
Page 6 lines 4,16,21-24,27=32	Spacing, text
Page 7 lines 2,6-7,9,12-16	Equation, spacing, text
Page 11 line 2	Correction to title
lines 7,19,22-25	Spacing, text
Page 12 lines 7,18,23-24,28,31-32	Spacing. Text
Page 13 lines 13-20	Format, explanatory text
Page 14 lines 9-14	Spacing
Page 15 lines 2-3,5,7-9,18,27	Spacing
Page 16 lines 4,7,11,15,20,29-31,34	Spacing
Page 17 line 2	Equations
Page 18 lines 10,14,16-17,20,24-27,31	Spacing
Page 19 lines 6,9,12,13-14,22,26,32	Spacing

Page 20 line 32	Redundant word
Page 21 lines 4,10,12,19-23,26,28-29	Spacing, remove "shaded"
Page 22 lines 3-6,8,13,19-20,23,25,31	Spacing, equation
Page 23 lines 30-33	Spacing
Page 24 lines 1,3-10,12,14-15,18-20,22,27,29-30	Spacing
Page 25 lines 1,3,5,7,9,13-14,16,24,26,28,29-30	Spacing, definitions of symbols
Page 26 lines 2-3,11-12,16,22-31	Spacing, corrections
Page 29 lines 4.12	Spacing
Page 30 line 31	Spacing
Page 31 lines 1,8,11,17,22	Spacing, corrections
Page 32 line 35	Correction
Page 33 lines 1-12,14,28-34	Corrections and text to explain non-factorable and factorable code matrices C from previous application 10/266,257 filed 10/08/2002
Page 34 lines 1-10	Continuation from previous
Lines 12-21	Redundant text
Page 35 lines 1-2,25-26,29,32,35	Spacing, corrections
Page 39 lines 3,8,12,14,22-23,26,35	Spacing, corrections

Page 40 lines 14-16,18-20,22-26,35    Spacing, corrections

Page 41 lines 5-6,9,29                    Spacing  
             Lines 12-26                    Text to explain FIG. 7B from  
   previous application  
   10/266,257 filed 10/08/2002

Page 42 lines 2,4,7,33                    Spacing

Page 44 lines 6-7,9,17-18,22,28    Spacing

Page 45 lines 3,5-12,22-24,26,29-31    Spacing

Page 45 lines 2-4,13,17-19,21,24,32,35    Spacing

Page 47 lines 8-34                        Deleted and placed in a separate  
   document

Pages 48-57                                Deleted and placed in a separate  
   document

Thanks ever for your guidance and help.

Sincerely,



Name                                    Urbain A. von der Embse  
E-mail                                    uavonderembse@ca.rr.com  
Contact No.                            310.641.0488  
Address                                   Urbain A. von der Embse  
    7323 W. 85<sup>th</sup> St.  
    Westchester, CA 90045-2444

APPLICATION NO. 10/806,016

INVENTORS: Urbain Alfred von der Embse

## TITLE OF THE INVENTION

Multi-scale code division frequency/wavelet multiple access

APPLICATION NO. 10/806,016

INVENTION: Multi-scale code division frequency/wavelet multiple  
access

**INVENTORS:** Urbain Alfred von der Embse

## CROSS-REFERENCE TO RELATED APPLICATIONS

### U.S. PATENT DOCUMENTS

US-5,761,341 A	June 1998	Go,Shiyu
US-6,144,773 A	Nov. 2000	Kolarov, et. al.
US-6,757,343 B1	June 2004	Ortega, et. al.
US-7,194,108 B2	March 2007	Tapson, Daniel Warren
US-7,277,488 B2	Oct. 2997	Tapson, Daniel Warren
US-7,295,695 B1	Nov. 2007	Dayal, Aditya
US-5,159,608	Oct. 1992	Falconer et. al.
US-6,885,852 B1	April 2006	Ozukturk et. al.
US-2002/0126,741	Sept 2002	Baum et.al
US-6,430,722	Aug 2002	Eroz et.al.
US-6,396,804	May 2002	Oldenwalder, Joseph P.
US-6,396,423	May 2002	Laumen et.al.
US-6,393,012	May 2002	Pankaj et.al.
US-6,389,138	May 2002	Li et.al.
US-6,366,624	April 2002	Balachandran et.al.
US-6,362,781	March 2002	Thomas et.al.
US-6,317,466	Nov 2001	Fuschini et.al.
US-6,317,413	Nov 2001	Honkasalo, Zhi-Chun
US-6,308,294	Oct 2001	Ghosh et.al.
US-6,239,767	May 2001	Rossi et.al.
US-6,167,079	Dec 2000	Kinnunen et.al.
US-6,160,854	Dec 2000	Heegard et.al.



## OTHER PUBLICATIONS

- [1] IEEE 802.11g standard
- [2] Application 09/826,118 filed on 01/09/2001 New multi-Resolution waveforms, U.A. von der Embse
- [3] Application 10/266,257 filed 10/08/2002 Multi-scale CDMA, U.A. von der Embse
- [4] Application 09/826,117 filed on 01/09/2001 Hybrid-Walsh codes for CDMA, U.A. von der Embse
- [5] "Multirate Digital Signal Processing", R.E. Crochiere, L.R. Rabiner, 1983, Prentice-Hall
- [6] "Multirate Systems and Filter Banks", R.P. Vaidyanathan, 1993, Prentice-Hall
- [7] "Wavelets and Filter Banks", Gilbert Strang, Truong Nguyen, 1996, Wellesley-Cambridge Press
- [8] Ronald R. Coifman, Yves Meyer, Victor Wickerhauser, "Wavelet analysis and signal processing", in "Wavelets and Their Applications", Jones & Bartlett Publishers, 1992
- [9] T. Blu, "A new design algorithm for two-band orthonormal rational filter banks and orthonormal rational Wavelets", IEEE Signal Processing, June 1998, pp. 1494-1504
- [10] M. Unser, P. Thevenaz, and A. Aldroubi, "Shift-orthogonal Wavelet bases", IEEE Signal Processing, July 1998, pp. 1827-1836
- [11] K.C. Ho and Y. T. Chan, "Optimum discrete Wavelet scaling and its application to delay and Doppler estimation", IEEE Signal Processing, Sept. 1998, pp. 2285-2290
- [12] I. Daubechies, "Ten Lectures on Wavelets", Philadelphia: SIAM, 1992
- [13] P.P. Vaidyanathan and T.Q. Nguyen, "Eigenvalues: A New Approach to Least-Squares FIR Filter Design and Applications Including Nyquist Filters", IEEE Trans. on

Circuits and Systems, Vo. CAS-34, No. 1, Jan. 1987, pp 11-23

- [14] J.H. Mc.Clellan, T.W. Parks and L.R. Rabiner, "A Computer Program for Designing Optimum FIR Linear Phase Filters', IEEE Trans Audio Electroacoust. Vol. AU-21, Dec. 1973, pp. 506-526

"

APPLICATION NO. 10/806,016

INVENTION: Multi-scale code division frequency/wavelet multiple  
access

INVENTORS: Urbain Alfred von der Embse

STATEMENT REGARDING FEDERALLY SPONSORED  
RESEARCH OR DEVELOPMENT

Not Applicable.

APPLICATION NO. 10/806,016

INVENTION: Multi-scale code division frequency/wavelet multiple  
access

INVENTORS: Urbain Alfred von der Embse

INCORPORATION-BY-REFERENCE OF MATERIAL  
SUBMITTED ON A COMPACT DISC

Not Applicable.

APPLICATION NO. 10/806,016

INVENTION: Multi-scale code division frequency/wavelet multiple  
access

INVENTORS: Urbain Alfred von der Embse

Marked up version of SUBSTITUTE SPECIFICATION

APPLICATION NO. 10/806,016

INVENTION: Multi-scale code division frequency/wavelet multiple  
\_\_\_\_\_ access

INVENTORS: Urbain Alfred von der Embse

5

This patent application is a continuation in part of  
application 09/826,118 filed on 01/09/2001, application  
09/526,117 filed on 01/09/2001, and application 10/266,257 filed  
on 10/08/2002.

10

## BACKGROUND OF THE INVENTION

### ~~TECHNICAL FIELD~~ I. Field of Invention

The present invention relates to both orthogonal frequency  
15 division multiple access OFDMA, to orthogonal Wavelet division  
multiple access OWDMA, to code division multiple access CDMA, and  
to multi-scale code division multiple access MS-CDMA, for  
cellular telephone and wireless data communications with data  
rates up to multiple T1 (1.544 Mbps), E1 (2.048 Mbps), Sonet,  
20 Ethernet, -and higher (>10 Gbps), and to optical CDMA and optical  
OWDMA. Applications are to wire, wireless local area, wide area,  
mobile, point-to-point, and satellite communication networks.  
More specifically the present invention relates to a new and  
novel means for combining MS-CDMA with OFDMA, to a new and novel  
25 OWDMA which is an orthogonal multi-resolution complex Wavelet  
multiple access generalization of OFDMA, and to a new and novel  
means for combining MS-CDMA with OWDMA. -This new architecture  
MS-CDMA OFDMA/OWDMA is an attractive candidate to replace current  
and future OFDMA applications and CDMA applications.

30

CONTENTS

BACKGROUND ART

page 1

<del>SUMMARY OF INVENTION</del>	<del>page 10</del>
<del>BRIEF DESCRIPTION OF DRAWINGS</del>	<del>page 12</del>
<del>DISCLOSURE OF INVENTION</del>	<del>page 13</del>
<del>REFERENCES</del>	<del>page 42</del>
5 <del>DRAWINGS</del>	<del>page 43</del>

## ~~BACKGROUND ART~~

### II. Description of Related Art

Current OFDMA art is represented by the applications to the  
 10 wireless cellular communications standards IEEE 802.11a, IEEE  
 802.11g, IEEE 802.15.3a, IEEE 802.16. -OFDMA uses the Fourier  
 transform basis vectors as the orthogonal channelization vectors  
 for communications with each basis vector multiplied by -a symbol  
 which is encoded with a data or pilot signal word.

15

The discrete Fourier transform DFT implemented as the fast  
 Fourier transform FFT is defined in equations (1). Step 1 defines  
 the digital sampling interval T over time, the sampling instants  
 $t=iT$  where i is the time index and where the sampling time  $1/T$  is  
 20 at least equal to the complex Nyquist sampling rate to prevent  
 spectral foldover. Step 2 is the FFT of the ~~time~~ complex  
baseband transmitted signal function  $Y_z(i)$  for the data block  
~~and step 3 is the inverse FFT<sup>-1</sup> of the frequency spectrum X(k)~~  
~~which derives the original time function Y(i). Step step 4-3~~  
 25 defines the NxN orthogonal complex DFT matrix E row vectors E(k)  
 which are the DFT harmonic vectors or basis vectors or code  
 vectors or channelization vectors. Step ~~5-4~~ is defines the  
~~complex baseband transmitted signal z(i) for one data block and~~  
 is equal to the inverse FFT transform  $FFT^{-1}$  of the user symbols  
 30  $x(k)$ .

Unweighted DFT encoding

(1)

1 Sampling interval of DFT

|-----|----- - - - - - |-----|

$$\begin{array}{ccccccc}
 t \rightarrow & 0 & T & . & . & . & (N-2)T & (N-1)T \\
 i \rightarrow & 0 & 1 & & & & N-1 & N-1
 \end{array}$$

where  $1/T \geq$  complex Nyquist sample rate

## 2 FFT of $\underline{Y}_Z(i)$

$$\begin{aligned}
 5 \quad X(k) &= \text{FFT}[\underline{Y}_Z(i)] \\
 &= \sum_i E(k, i) \underline{Y}_Z(i) \\
 &= \sum_i \exp(-j2\pi ki/N) \underline{Y}_Z(i)
 \end{aligned}$$

## ~~3 FFT<sup>-1</sup> of X(k)~~

$$\begin{aligned}
 10 \quad \underline{Y}(i) &= \text{FFT}^{-1}[\underline{X}(k)] \\
 &= N^{-1} \sum_k E^*(k, i) \underline{X}(k) \\
 &= N^{-1} \sum_k \exp(j2\pi ki/N) \underline{X}(k)
 \end{aligned}$$

## 43 DFT orthogonal harmonic code matrix E

$$\begin{aligned}
 15 \quad E &= N \times N \text{ DFT orthogonal DFT matrix} \\
 &= [E(k, i)] \text{ matrix of elements } E(k, i) \\
 E(k, i) &= \exp(-j2\pi ki/N) \text{ harmonic } k, \text{ time index } i \\
 E(k) &= \text{harmonic } k \text{ basis (code) vector} \\
 &= [E(k, 0), E(k, 1), \dots, E(k, N-1)] \\
 EE^* &= N I \\
 20 \quad \langle E(k), E^*(k') \rangle &= \delta(k-k') N I \\
 \text{where } I &= N \times N \text{ identify matrix} \\
 E^* E^* &= \text{complex conjugate transpose of } E \\
 \delta(k-k') &= \text{Dirac delta function} \\
 &= 1 \text{ for } k=k' \\
 25 \quad &= 0 \text{ otherwise}
 \end{aligned}$$

## 54 Transmitted DFT complex baseband signal $z(i)$ for one data block

$$\begin{aligned}
 30 \quad d(k) &= \text{data modulation for user } k \\
 &= \text{encoded amplitude } A(k) \text{ and phase } \phi(k) \\
 x(k) &= \text{transmitted symbol encoded with } d(k) \\
 &= A(k) \exp(j\phi(k)) \\
 z(i) &= \text{FFT}^{-1}[x(k)]
 \end{aligned}$$



$$= N^{-1} \sum_k x(k) E^*(k, i)$$

OFDMA for IEEE 802.11g in reference [1] is illustrated in  
 5 FIG. 1. The channelization filter  $h(f)$  -1 covers a 20 MHz  
 frequency band 2 assigned to OFDMA. Plotted is the power spectral  
 density  $PSD = |h(f)|^2$  of this channelization filter  $h(f)$ . -A  $N=64$   
 point fast Fourier transform FFT covers this band 2. -Consistent  
 with the IEEE specification, -FIG. 1 refers to the DFT which is  
 10 identical to the analog Fourier transform FT since it is the  
 sampled data version of the FT. It is convenient to consider the  
 DFT in this invention disclosure as the digital format for the  
 FFT. The DFT frequency spectrum 3 consists of  $N=64$  equally spaced  
 filters 4 across this 20 MHz band. -Filter spacing is equal to  
 15 the DFT output rate  $1/NT = 0.3125 \text{ MHz} = 20\text{MHz}/64$ . The DFT time  
 pulse  $p(t)$  5 is  $NT=3.2\mu\text{s}$  in length and the total DFT period -6 is  
 $4.0\mu\text{s}$  which allows a  $0.8\mu\text{s}$  guard time for  $p(t)$ .

Throughout this invention disclosure it will be understood  
 20 that the FFT fast algorithm will always be used to implement the  
 DFT and the inverse  $FFT^{-1}$  fast algorithm will always be used to  
 implement the inverse  $DFT^{-1}$ .

OFDMA transmitter encoding of the OFDMA waveform in FIG.1  
 25 is defined in equations (2). Step 1 lists the parameters and  
 definitions and step 2 defines the time domain weighting. Step 3  
 is the complex baseband transmitted signal  $z(i)$ .

OFDMA encoding for transmitter (2)

30 1 Parameters and definitions from 1 in equation (1)  
 and

$h(i)$  = 20 MHz band filter impulse response  
 $p(i)$  = impulse response of the DFT waveform  
 = real weighting function in 6 in FIG. 1

$N = 64$  point DFT  
 $1/T = 30\text{--}20$  MHz sample rate for DFT  
 $\geq$  complex Nyquist rate  
 $NT = 3.2\mu\text{s}$  DFT length  
 $1/NT = 0.3125$  MHz DFT output rate  
 $=$  DFT channel separation  
 $=$  DFT tone spacings  
 52 channels are used: 4 pilot, 48 data  
 12 guard band channels for rolloff of the  $h(k)$

## 2 Pulse $p$ and band filter $h$ weighting for DFT basis vectors

$\underline{p} = p \otimes h$ , convolution of  $p$  and  $h$   
 $=$  filter transfer function in time domain  
 for the combined  $h$  and  $p$  filters  
 $= [ \underline{p}(0), \underline{p}(1), \dots, \underline{p}(N-1) ]$

## 3 Transmitted OFDMA encoded baseband signal $z(i)$ for one data block

$d(k) =$  data modulation for user  $k$   
 $=$  encoded amplitude  $A(k)$  and phase  $\varphi(k)$   
 $x(k) =$  transmitted symbol encoded with  $d(k)$   
 $= A(k) \exp(j\varphi(k))$   
 $z(i) = \text{FFT}^{-1}[ \underline{p}(i) x(k) ]$   
 $= N^{-1} \sum_k \underline{p}(i) x(k) E^*(k, i)$

OFDMA for IEEE 802.11g has the strict orthogonality of the DFT(FFT) –replaced by cross-correlations between the 48 channel tones and other impacts due to the band channelization and pulse weighting  $p \otimes h$  plus the time errors  $-\Delta t$  and frequency errors  $\Delta f$  from synchronization errors, multi-path, propagation, and terminal stresses. –These impacts on orthogonality are low enough to allow OFDMA to support higher values for the symbol signal-to-noise ratio  $S/N$  in the detection band that are required for

higher order symbol modulations. The highest order symbol modulation currently is 64 state quadrature amplitude modulation 64-QAM corresponding to 6 bits per symbol where  $6 = \log_2(64)$  and  $\log_2(o)$  is the logarithm to the base 2. With rate  $-3/4$  convolutional coding the highest information rate is 4.5 bits/symbol =  $6 \times 3/4$ . Required S/N at a BER=1.0e-6 is approximately S/N~19 dB.

OFDMA for IEEE 802.11g provides 48 channels over a 20 MHz frequency band at a symbol rate equal to 0.25 MHz =  $1/4.0 \mu s$  from 6 in FIG. 1 and for a maximum information rate equal to 4.5 bits/symbol this equals a burst rate of 54 MBps =  $4.5 \times 48 \times 0.25$ . Some spread spectrum properties are realized by hopping the 20 MHz band, shuffling the channel assignments over the 48 available channels for a user assigned to several channels in order to spread his transmissions over the band, -and for "flash" OFDMA by a random hopping of each user channel across the 48 available channels within the band.

OFDMA receiver decoding of the OFDMA waveform in FIG.1 is defined in equations (3) for the receiver. ~~Step 1 identifies the parameters and definitions and defines the convolution  $R(i) = p \odot p$  of the time domain weighting  $p$  with itself in the matched filter receiver. Step 2 and~~ derives the estimates of the transmitted symbols by implementing matched filter detection in the receiver.

OFDMA decoding for receiver (3)

~~1 Parameters and definitions are defined in 1,2 in equations (2) together with~~

~~$R(i) = \text{convolution of } p \text{ with } p$~~   
 ~~$= p \odot p$~~

2-OFDMA decoding for one data block derives estimates

$\hat{x}(k)$  of  $x(k)$  from the receiver estimates  $\hat{z}(i)$  of  $z(i)$

$$\hat{x}(k) = \text{FFT}[ \hat{z}(i) \odot p ]$$

$$\begin{aligned} &= \sum_i \hat{z}(i) \odot p_E(k, i) \\ &\underline{\hspace{1cm}} \cong \underline{x(k)} \end{aligned}$$

Current CDMA spread spectrum art is illustrated by the waveform in FIG. 2 which describes the waveform for full band CDMA communications over the band  $-B$  ~~9~~ which is the output range ~~8~~ of the band filter  $h(f)$  ~~7~~. The ~~CDMA~~ chip rate  $1/T_c$  ~~12~~ is less than the available frequency band  $B$  to allow the chip frequency spectrum  $p(f)$  ~~10,11~~ to roll off. ~~As defined 9 in~~ FIG. 2 the band  $B$  and chip rate are related by equation  $B=(1+\alpha)/T_c$  where  $\alpha$  is the bandwidth expansion factor close to  $\alpha=0.25$  for high performance communications. ~~Frequency spectrum  $p(f)$  10 for the CDMA communications is essentially equal to the Fourier transform of the CDMA time pulse  $p(t)$  13. The representative~~ time pulse  $p(t)$  is a square-root raised cosine pulse which ~~is~~ can typically ~~be~~ used for high performance communications to obtain a reasonably flat spectrum with a sharp rolloff at the edges to enable the chip rate  $1/T_c$  to be reasonably close to the available frequency band  $B$ .

Chip rate  $1/T_c$  is the CDMA total symbol rate. The users could be at different data rates but this and other architectural variations do not limit the scope of this invention. Power is uniformly spread over the CDMA pulse waveform spectrum  $p(f)$ .

It is self evident to anyone skilled in the CDMA communications art that these communications mode assumptions are both reasonable and representative of the current CDMA art and do not limit the applicability of this invention.

CDMA encoding of the waveform in FIG. 2 for the transmitter is defined in equations (4). Steps 1,2 define the CDMA transmission and parameters. Step 3 defines the user symbol  $x(u)$ . Step 4 is the set of Walsh orthogonal channelization codes

$w(u)$  and step 5 is the pseudo-random PN covering or spreading code. Step 6 defines the complex baseband signal  $z(t)$  as the waveform  $p(t-nT_c)$  multiplied by the encoded sum over  $u$  and  $n$ .

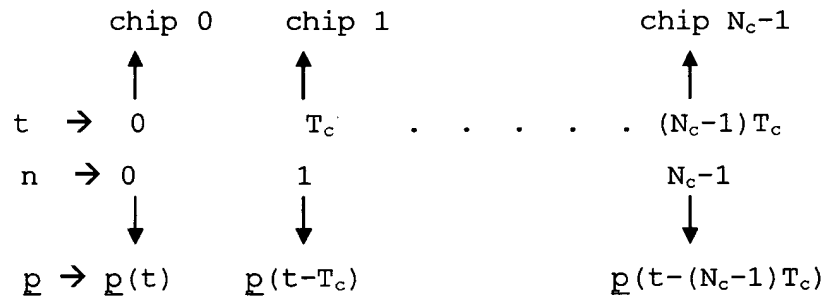
5

CDMA encoding for transmitter

(4)

1 CDMA transmission

10



15

where  $p(t-nT_c) = p(t-nT_c) \odot h(t)$   
 $=$  convolution of  $p(t-nT_c)$  and  $h$   
 $=$  filter transfer function in time domain  
for the combined  $p(t-nT_c)$  and  $h$  filters

20

2 Parameters and definitions

$N_c$  = Number of users and orthogonal code chips  
 $T_c$  = CDMA chip length or repetition interval  
 $1/N_c T_c$  = User symbol rate

25

3 User complex signal  $x(i)$

$d(u)$  = data modulation for user  $u$   
 $=$  encoded amplitude  $A(u)$  and phase  $\phi(u)$   
 $x(u)$  = transmitted symbol encoded with  $d(u)$   
 $= A(u) \exp(j\phi(u))$

30

4 Walsh orthogonal channelization code matrix  $W$

$W$  = Code matrix,  $N_c$  rows of  $N_c$  code vectors

$= [ W(k,n) ]$  matrix of elements  $C(k,n)$   
 $W(u,n) = +/-1$ , chip  $n$  of vector  $u$   
 $W(u)$  = code vector  $u$ , row  $k$  of  $W$   
 $= [W(u,0), W(u,1), . . . , W(u,N_c-1)]$

5

5 PN covering (spreading) code  $P(n)$  for chip  $n$   
 $P(n) = \exp(j\phi(n))$

6 Transmitted CDMA complex baseband signal  $z(t)$

10  $z(t) = N_c^{-1} \sum_u \sum_n P(n) W(u,n) x(u) p(t-nT_c)$

CDMA decoding of the waveform in FIG. 2 for the transmitter  
 is defined in equations (5). -Step 1 defines the convolution  
 15  $R(n,n-n')$  of the CDMA pulse waveform with itself in the matched  
 filter receiver. Steps 2,3 are the Walsh and PN decoding  
 properties. Step 4 uses the matched filter detection -theorem to  
 derive the estimates of the transmitted symbols.

20 CDMA decoding for receiver (5)

1 Parameters and definitions are defined in  
 1,2,3 in equations (4) together with

$R(n,n-n') =$  convolution of  $p(t-nT_c)$  with  
 $p(t-n'T_c)$  evaluated at the receiver

25 detection times  $t=nT_c$

$= R(n) \delta(n-n')$   
 $= R(n) \quad \text{for } n=n'$   
 $= 0 \quad \text{otherwise}$

30 2 Walsh decoding of channelizaation codes  $W(k)$

$WW^* = N_c I$   
 where  $I = N_c \times N_c$  identify matrix  
 $W^* =$  conjugate transpose of  $W$   
 $\langle W(k), W(k') \rangle = N \delta(k-k')$

3 PN decoding

$$P(n)P^*(n) = 1 \text{ for all } n$$

where  $P^*$  = complex conjugate of  $P$

5

4 CDMA decoding

$$\hat{x}(k) = \sum_n P^*(n)W^*(k,n) \hat{z}(t) \odot p(t-nT_c)$$

10 It should be obvious to anyone skilled in the communications art that these example implementation algorithms in equations (1),(2),(3),(4),(5) clearly define the fundamental OFDMA and CDMA signal processing relevant to this invention disclosure and it is obvious that this example is representative of the other possible signal processing approaches.

15

For cellular applications the encoding algorithms for the transmitter describe the implementation of OFDMA and of CDMA encoding and are the transmission signal processing applicable to this invention for both the hub and user terminals, and the  
20 decoding algorithms for the receiver describes the corresponding OFDMA and CDMA receiving signal processing for the hub and user terminals for applicability to this invention.

For optical communications applications the microwave  
25 processing at the front end of both the transmitter and the receiver is replaced by the optical processing which performs the complex modulation for the optical laser transmission in the transmitter and which performs the optical laser receiving function of the microwave processing to recover the complex  
30 baseband received signal with the remainder of the signal processing functionally the same for the OFDMA and for the CDMA encoding transmitter and functionally the same as described for the OFDMA and CDMA receiving signal processing receiver.

## BRIEF SUMMARY OF THE INVENTION

This invention introduces the new OWDMA communications  
5 technology which implements orthogonal multi-resolution complex  
Wavelet division -multiple access and is a multi-resolution  
complex Wavelet generalization of OFDMA; introduces the new  
application of the multi-scale code division multiple access MS-  
CDMA architecture which integrates MS-CDMA with OFDMA and with  
10 OWDMA, -and introduces the variable gain control over frequency.

The new OWDMA forms a uniform set of contiguous orthogonal  
filters across the available frequency band in one of the  
numerous available architectural options and which -implements a  
15 polyphase filter bank across the available frequency band with  
the basic property that the filters are orthogonal. Each filter  
defines a OWDMA channel for communications. Similar to OFDMA,  
the symbol rate within each channel is equal to the channel  
separation  $1/NT$  where  $N$  is the number of channels,  $1/T$  is the  
20 frequency band, and  $NT$  is the symbol-to-symbol separation and  
which is made possible by using multi-resolution complex Wavelet  
channelization filters developed in -reference [2]. -OWDMA  
filters are orthogonal in frequency which means their frequency  
spectrums are non-overlapping, -and they have flat spectrums  
25 across ~~the~~ each channel.

OWDMA orthogonality only requires frequency synchronization  
whereas OFDMA orthogonality requires both time and frequency  
synchronization which means that time synchronization errors on  
30 the user communication channels for the return links to the hub  
or access point for cellular communications does not degrade the  
orthogonal separation between the channels as they will for  
OFDMA.



Sensitivity to frequency synchronization errors is less for OWDMA primarily because of the non-overlapping of the frequency spectrums. -The overall tolerance to user-to-user imbalances on the return communications channels is better for OWDMA primarily because their design keeps the frequency spectrums from overlapping in the presence of real operational conditions with synchronization errors. —This allows the forward communications link to support a power imbalances between channels to mitigate differing ranges and path losses at the user antennas.

Application of multi-scale code division multiple access MS-CDMA in reference [2] provides a means to implement the new hybrid-Walsh orthogonal CDMA codes in reference [4] over the OFDMA/OWDMA channels by spreading the CDMA within each channel and over all of the channels such that each user can be -spread over the complete band. This keeps the chip rate equal to  $1/NT$  while maintaining the spreading over the fullband  $1/T$  and allows the band transmit Tx power to be independently controlled in frequency. These two CDMA scales  $1/NT, 1/T$  are generated by MS-CDMA in combination with the OFDMA/OWDMA channelization filter banks. The  $1/T$  scale is to combat fading and interference similar to the current CDMA, and the  $1/NT$  scale is for acquisition, synchronization, and equalization and protection against multipath and -provides the flexibility for band power control to provide a frequency diversity communications improvement.

Variable control over the frequency B can be implemented by partitioning the -CDMA over the channels into separate groups and assigning an independent power level to each group of channels. OWDMA readily supports differences in power levels between adjacent channels. -Power control is desirable to support differences in -quality of service SoC, range losses-, and path loss.

It should be obvious to one familiar with the CDMA communications art that the number of scales could be a larger number than the two used in this invention disclosure and the multi-resolution complex Wavelet design for OWDMA supports the partitioning of the frequency band  $1/T$  into several frequency scales for the channelization filters, supports individual multi-resolution complex Wavelet packets for communications in the time-frequency domain and which can be integrated into the MS-CDMA, -and supports separate and segmented communications bands simultaneously.

#### BRIEF DESCRIPTION OF SEVERAL VIEWS OF THE DRAWINGS

The above-mentioned and other features, objects, design algorithms, and performance advantages of the present invention will become more apparent from the detailed description set forth below when taken in conjunction with the drawings and performance data wherein like reference characters and numerals denote like elements, and in which:

FIG. 1 is a description of the OFDMA waveform in the frequency and time domains over a 20 MHz band for the IEEE 802.11g standard.

FIG. 2 is a description of the CDMA waveform in the frequency and time domains over a band B.

FIG. 3 is a description of the OWDMA waveform in the frequency and time domains over a band B.

FIG. 4 is a description of the design requirements on the multi-resolution complex Wavelet  $PSD = |\psi(f)|^2$ .

FIG. 5 is a representative baseband frequency response for the multi-resolution complex Wavelet filter and the square-root cosine filters for the bandwidth expansion factors =0.22, 0.40.

5

FIG. 6 is a representative implementation of MS-CDMA OFDMA/OWDMA encoding for transmitter.

FIG. 7A is a representative MS-CDMA OFDMA/OWDMA transmitter  
10 implementation block diagram. ~~for the MS-CDMA OFDMA/OWDMA~~  
~~transmitter.~~

FIG. 7B is a representative MS-CDMA OFDMA/OWDMA transmitter  
15 MS-CDMA mapping.

FIG. 8 is a representative implementation of MS-CDMA OFDMA/OWDMA encoding for the receiver.

FIG. 9 is a representative implementation block diagram for the MS-CDMA OFDMA/OWDMA receiver.

20

## DISCLOSURE-DETAILED DESCRIPTION OF THE INVENTION

This invention introduces the new orthogonal multi-resolution complex Wavelet division multiple access OWDMA  
25 communications technology which is a multi-resolution complex Wavelet generalization of OFDMA, introduces the new multi-scale code division multiple access MS-CDMA architecture to integrate MS-CDMA with OFDMA, and introduces the new MS-CDMA architecture  
30 to integrate MS-CDMA with OFDMA,

OWDMA uses the multi-resolution complex Wavelet waveform developed in -reference [2] to generate multi-rate orthogonal filter banks and waveforms to support communications at the

critical symbol rate equal to a combined  $1/T$  symbols per second for a  $1/T$  Hz frequency band. —The critical symbol rate is the Nyquist sample rate and —corresponds to no excess bandwidth  $\alpha=0$  in the equation  $B=(1+\alpha)/T$  which can be rewritten as the

5 bandwidth-time product  $BT=(1+\alpha)$  that relates filter bandwidth  $B$  and the symbol rate  $1/T$  for communications supported by this filter. —For the OFDMA filter —in —FIG. 1 —it is observed  $BT=1+\alpha=64/52=1.33$  —corresponding to  $\alpha=0.33$  —since 52 of the 64 DFT filters generated are used and for the —CDMA filter in FIG. 2  
10 it is observed that the bandwidth-time product is about  $BT=1.25$  corresponding to  $\alpha=0.25$  for the 3G CDMA cellular communications systems.

Multi-resolution complex Wavelet design algorithms were  
15 developed in reference [2] as a means to design polyphase multirate filters, quadrature mirror filters (QMF), perfect reconstruction filters, Wavelet iterated filter banks, and Wavelet tiling of the time-frequency  $t$ - $f$  space. —Prior to the invention of multi-resolution complex Wavelet design algorithms,  
20 theoretical studies had not yielded useful realizable filters for system applications implementing these architectures as summarized by the digital filtering and polyphase research in references [5],[6] and the Wavelet research as summarized in the references [7],[8],[9],[10],[11].

25

Multi-resolution complex Wavelets defined in equations (6), (7),(8) expand the Wavelet analytical formulation —to include a frequency variable which specifies the center frequency of our new Wavelets and performs a frequency translation of the real  
30 mother Wavelet to this center frequency. Currently, Wavelets are real functions of the scale and translation parameters. Multi-resolution complex Wavelets are functions of these parameters plus the frequency variable. This —new concept of frequency as an additional parameter provides an added degree of flexibility and

together with the Fourier domain design approach provide an entirely new means for deriving these new waveforms as generalization of the traditional real Wavelets and their modification to include the added frequency parameter. -With  
 5 frequency translation as the additional parameters the analytical formulation of these new waveforms as a function of the baseband or mother -waveform centered at dc( dc refers to the origin  $f=0$  of the frequency space) is given in equations (6).

10 Equations (6) introduce the new multi-resolution complex Wavelets. Step 1 -in equations (6) is the definition for the continuous real Wavelet over the time-frequency t-f space from reference [7],[9],[12] where  $\psi(t)$  is the Wavelet which is a waveform of finite extent in time t and frequency f over the t-f  
 15 space. Wavelet parameters a,b -are the Wavelet dilation and translation respectively or equivalently are the scale and shift. The  $\psi(t)$  without the indices a,b is the mother Wavelet which is a real and symmetric localized function in the t-f space used to generate the doubly indexed Wavelet  $\psi(t|a,b)$  where  $\psi(t|a,b)$  reads  
 20 the  $\psi$  is a function of time t for the parameters a,b. -Scale factor  $|a|^{-1/2}$  has been chosen to keep the norm of the Wavelet invariant under the parameter change a,b. Norm is the square root of the energy of the Wavelet response. The Wavelets  $\psi(t|a,b)$  and  $\psi(t)$  are localized functions in the t-f space which means that  
 25 both their time and frequency lengths are bounded.

Step 2 in equations (6) defines the digital Wavelet which is a Wavelet over the digital t-f space corresponding to digitization of the t-f space at -the  $1/T$  analog-to-digital A/D  
 30 rate where T is the interval between digital samples and " $i^L$ " is the index over the digital samples. -Time "t" for the continuous Wavelet in 1 is replaced by the equivalent digital sample number "i" corresponding to  $t=iT$  at sample i. -Wavelets in digital t-

f space have an orthogonal basis that is obtained by restricting the choice of the parameters a,b to the values  $a=2^{-p}$ ,  $b=qN2^p$  where p,q are the new scale and translation parameters and N is

5 New multi-resolution complex Wavelets (6)

1 Current continuous Wavelet as a function of the mother Wavelet at dc

$$\psi(t|a,b) = |a|^{-1/2} \psi((t-b)/a)$$

where a,b are the dilation,shift parameters

10

2 Current digital real Wavelet

Digital Wavelet shift q and scale p

$$a = 2^p$$

$$b = q N 2^p$$

15

Digital Wavelet as a function of mother Wavelet

$$\psi(i|p,q) = 2^{-p/2} \psi(2^{-p} i - qN)$$

where N = number of samples over the Wavelet spacing or repetition interval

$$T_s = NT \text{ wavelet spacing}$$

20

$$i = \text{digital sampling index}$$

$$T = \text{digital sampling interval}$$

$$t = iT \text{ time index at digital sample } i$$

3 Our new digital multi-resolution complex Wavelet is a function of the mother Wavelet at dc

25

3.a) Generalized complex format

$$\psi(i|p,q,k) = 2^{-p/2} \psi(2^{-p} i - qN) \exp[j2\pi f_c(p,k) 2^{-p} iT]$$

3.b) Multi-rate filter complex format

$$\psi(i|p,q,k) = 2^{-p/2} \psi(2^{-p} i - qN) E^*(k2^p, 2^{-p} i)$$

30

4 Orthogonality equations for our new digital complex Wavelet

$$\begin{aligned} \sum_i \psi(i|p,q,k) \psi^*(i|p',q',k') &= N \text{ iff } p=p', q=q', k=k' \\ &= 0 \text{ otherwise} \end{aligned}$$

5 New digital complex Wavelet for a uniform filter bank  
as a function of the mother Wavelet at dc

$$\psi(i|p=0,q,k) = \psi(i-qN) E^*(k,i)$$

5 the spacing or repetition interval  $T_s=NT$  of the Wavelets (which  
from a communications viewpoint are symbols) at the same scale  $p$ .  
Wavelets at  $p,q$  are related to the mother Wavelet by the equation  
in 2 —where the mother Wavelet is a real and even function of the  
sample coordinates and which follows directly from 1 for the  
10 continuous t-f space—.

Steps 3,4 in equations (6) define our new Wavelet in the  
digital t-f space and their orthogonality properties. Our new  
Wavelets are complex generalizations of —Wavelets in t-f space  
15 which enable them to be useful for communications and radar  
applications. —This generalization is accomplished —1) by the  
addition of a frequency translation parameter  $k$  which controls  
the frequency offset of the Wavelet, 2) by generalizing the  
Wavelet weighted orthonormality properties in step 4 to apply to  
20 waveforms over the time translation  $-q$  and over the scales  $p$  with  
the inclusion of the frequency translation and where  $\psi^*$  is the  
complex conjugate of  $\psi$  and is required in the orthogonality  
equations since the multi-resolution complex Wavelet  $\psi$  becomes  
complex with the addition of harmonic  $k$ , and —3) by their  
25 characterization and design in the Fourier domain. —The frequency  
parameter  $k$  controls the frequency translation  $-\exp[j2\pi f_c(p,k)2^{-p}iT]$  —in 3 for the generalized format and the frequency  
translation  $E^*(k2^p,2^{-p}i)$  in 3 for the multi-rate filter format.  
With this frequency translation the analytical formulation 3 of  
30 these new Wavelets is given as a function of the baseband or  
mother —waveform centered at dc corresponding to  $k=0$ . —Purpose of  
the frequency index  $k$  is to identify the center frequencies of  
the waveforms at the scale  $p$  and time translation  $q$  in the t-f  
space. The generalized center frequency  $f_c(p,k)$  of the frequency

translated dc waveform at scale  $p$  and frequency index reduces to  $f_c(p,k) = k2^p/NT$  for application to multi-rate filters.

Step 5 in equations (6) defines the equations for our new Wavelet for a uniform polyphase filter bank which is one of several OWDMA candidate architectures. —Our new Wavelets in 5 are the impulse responses of the corresponding digital symbols encoded with digital data for transmission which is the synthesis filter bank in polyphase theory, —and are the filter bank detection impulse responses for recovery of the transmitted digital symbols by the analysis filter bank in polyphase theory assuming matched filter detection. —The digital filters are observed to be the DFT's of the mother wavelets similar to the construction of the OWDMA in ~~equations 3~~ inequations (2) upon replacing the OWDMA pulse waveform  $p$  —with the mother Wavelet  $\psi(i-qN)$  in 5 and setting  $q=0$  corresponding to the symbol at  $t=0$ .

Equations (7) derive the multi-resolution complex Wavelet as a function of the Fourier design coordinates. Design algorithms provide a means to design the mother Wavelet in the Fourier frequency domain to fit the communications and radar specifications. —From this mother Wavelet,—the Wavelets at the appropriate scales  $\{p,q,k\}$  are easily found as demonstrated in 3,5 in equations (6). Design in the frequency domain means the design coordinates specifying the Wavelet are Fourier frequency harmonics or coordinates. —Step 1 lists the parameters and coordinates and step 2 defines the Fourier harmonic frequency design coordinate.

Step 3 —is the DFT representation from equations 3 in equations (1) of the real mother Wavelet  $\psi(i)$  as a function of the DFT Fourier harmonic coefficients  $\{-\{\psi(k')\}\}$ . —Step 4 derives the equation for the multi-resolution complex Wavelet as a



function of the DFT Fourier harmonic coefficients by substituting  
 3 in equations (7) into 3 in equations (6).

Digital multi-resolution complex Wavelet is a (7)

function of the Fourier harmonic design coordinates

1 Parameters and coordinates

$\psi = \psi(i)$  mother Wavelet

$= \psi(i|p=0, q=0, k=0)$

$N' =$  length of the mother Wavelet  $\psi(i)$

$= NL+1$

$N =$  sampling interval of  $\psi$

$=$  spacing of  $\psi$  for orthogonality

$L =$  length of  $\psi$  in units of  $M-N$

$T =$  spacing of digital samples

$T_s = N'T$  length of  $\psi$  in seconds

2 Fourier harmonic frequency design coordinates

$\psi(k') =$  value of the Fourier design harmonic  $k'/N'T$

$=$  Fourier harmonic design coordinate value

$\{k'\} =$  set of harmonic design coordinates

for  $k' = 0, +/-1, . . .$

corresponding to harmonics  $\{k'/N'T\}$

3 Multi-resolution real mother Wavelet definition

$\psi(i) = \sum_{k'} \psi(k') E^*(k', i)$

4 Multi-resolution complex Wavelet definition

$\psi(i|p, q, k)$

$= 2^{-p/2} \sum_{k'} \psi(k') E^*(k'/L+k2^p, 2^{-p}(i-qN))$

Several fundamental properties follow directly from the  
 frequency design approach in ~~equations 4~~ in equations (7). It is  
 demonstrated in reference [2] that our multi-resolution complex

Wavelets are implemented with our design in the Fourier domain and our multi-resolution complex Wavelet design remains invariant under scale changes. It is demonstrated in reference [2] that the Fourier frequency domain design in ~~equations 3~~ in equations (7) remains invariant for all parameter changes and in particular for all scale changes.

Multi-resolution complex Wavelet design algorithms are illustrated by a representative least-squares LS design for a OWDMA polyphase uniform filter bank in FIG. 3. There are two categories of LS algorithms and these are the eigenvalue and the gradient search that respectively can be reduced to algorithms equivalent to the original eigenvalue [13] and Remez-exchange [14] waveform design algorithms for application to a uniform filter bank.

FIG.3 is an example of an orthogonal multi-resolution complex Wavelet division multiple access OWDMA Wavelet polyphase filter bank over the band B by setting N in 5 in equations (6) equal to the N in 17 in FIG. 3. The band channelization filter is a roofing filter  $h(f)$  that covers the B frequency band assigned to OWDMA. Unlike its use in FIG. 1 for the OWDMA with B=20 MHz this filter is intended to be flat over the frequency band B of interest and with a bandwidth equal to  $1/T > B$ . Plotted is the power spectral density  $PSD=|h(f)|^2$  of this channelization filter  $h(f)$ . The  $h(f)$  output is digitized at the sample rate  $1/T$  to form the OWDMA multi-resolution complex Wavelet polyphase filter bank which are uniformly spaced at  $1/NT$  Hz over the  $1/T$  frequency band  $1/T$ . This digital sample rate  $1/T$  is sufficiently large to allow the use of the shaded OWDMA filters in 17 in FIG. 3 for communications over the band B with no excess bandwidth  $\alpha=0$  unlike the OFDMA in FIG. 1 which has  $\alpha=0.33$  and CDMA in FIG. 2 with  $\alpha=0.25$ .

This OWDMA polyphase filter bank is ideally decimated which means the filter output multi-resolution complex Wavelet sample rate  $1/T_s$  is equal to the channel-to-channel spacing  $1/T_s = 1/NT$  equivalent to stating that there is no excess bandwidth  $\alpha=0$  within the filter bank. A representative OWDMA multi-resolution complex Wavelet  $\psi$  for  $L=8$  is plotted in — as a function of the time offset expressed in units of the multi-resolution complex Wavelet spacing  $NT$ . This  $\psi$  was designed by the eigenvalue category of LS design algorithms. Our design for this topology is immediately applicable to an arbitrary set of multi-resolution OWDMA filters through the scaling equations—3,4 in equations (7) which gives the design of our Wavelet at arbitrary scales—in terms of our design of the mother Wavelet.—.

Multi-scale mother Wavelet  $\psi$  design for the OWDMA polyphase filter bank in FIG. 3 starts by using the frequency design template in FIG. 4 to construct the LS error metrics as a function of the DFT frequency design coordinates  $\{\psi(k')\}$  in 2 in equations (7) which define the mother Wavelet in —3 in equations (7). Next minimization search algorithms are developed and used to find the values of  $\{\psi(k')\}$  that minimize the weighted sum of these LS design error metrics equal to the cost function  $J$  in equations (8) for the LS design. Minimizing  $J$  with respect to the  $\{\psi(k')\}$  gives the design values of  $\{\psi(k')\}$  for constructing the mother Wavelet in equations—3 in equations (7).

Equations (8) define the LS cost function  $J$  constructed with the LS metrics with the aid of the frequency design template in FIG. 4. Step 1 defines the LS error metrics which are the passband error metric  $\mu(1)$ , stopband error metric  $\mu(2)$ , quadrature mirror filter QMF error metric  $\mu(3)$ , intersymbol interference error metric  $\mu(4)$ , and the adjacent channel interference error metric  $\mu(5)$ .

LS cost function J for designing  $\psi(i)$  (8)

1 LS metrics

- 5  $\mu(1)$  = passband LS error metric measures the LS  
error of the passband ripple -24 in FIG.4
- $\mu(2)$  = stopband LS error metric measures the  
stopband atteunation 28 in FIG. 4
- 10  $\mu(3)$  = quadrature mirror filter QMF LS error  
metric measures the flatness of the sum  
of two contiguous filters over the  
deadband 31 in FIG. 4
- $\mu(4)$  = orthogonality LS error metric measures the  
intersymbol interference ISI between  
overlapping Wavelets of different wymbol
- 15  $\mu(5)$  = orthogonality LS error metric measures the  
adjacent channel interference ACI from  
nearest neighbor channels
- 2 Metric weighting
- 20  $w(n)$  = metric weight for error metrics  $n=1,2,3,4,5$  in 1  
 $\geq 0$   
such that  $\sum_n w(n) = 1$
- 3 Cost function J
- 25  $J = \sum_n w(n) \mu(n)$

FIG. 4 is the frequency design template for the power spectral density  $PSD=|\psi(f)|^2$  of the multi-resolution complex Wavelet and defines the parameters of interest for the passband metric  $\mu(1)$  -and stopband metric  $\mu(1)$  in step 1 in equations (8). Passband 21 -of the wavelet  $PSD=|\psi(f)|^2$  -is centered at dc ( $f=0$ ) since we are designing the mother Wavelet, -and extends over the frequency range  $\omega_p$  extending from  $-\omega_p/2$  to  $+\omega_p/2$  -22 -in units of

the radian frequency variable  $\omega = 2\pi fT$  —23— where  $T$  is the digital sampling interval defined in FIG. 3. The frequency space extends over the range of  $-\Delta f = -1/2T$  to  $\Delta f = +1/2T$  —which is the frequency range in FIG. 3 and the mother Wavelet is at the center of the frequency band. —Quality of the  $PSD = |\psi(f)|^2$  —over the passband is expressed by the passband ripple —24. —Stopband 25 starts at the edge —26— of the passbands of the adjacent channels  $\pm \omega_a/2$  —26— and extends to the edge of the frequency band  $\omega = \pm \pi$  —27— respectively. —Stopband attenuation —28 at  $\pm \omega_a/2$  measures the  $PSD = |\psi(f)|^2$  —isolation between the edge of the passband for the mother Wavelet and the start of the passband for the adjacent Wavelet channel centered at  $\pm \omega_s$  —29. —Rolloff —30 of the stopband is required to mitigate the spillover of the Wavelet channels other than the adjacent Wavelet channels, —onto the mother Wavelet channel. —Deadband or transition band 31 —is the interval between the passbands of contiguous Wavelet channels, and is illustrated in FIG. 4 by the interval from  $\omega_p/2$  to  $\omega_a/2$  —between the mother Wavelet channel and the adjacent Wavelet channel at  $\omega_a$ . —Waveform sample rate  $\omega_s$  —32— is the waveform repetition rate. —For the LS example algorithm, —the waveform sample rate is equal to the channel-to-channel spacing for zero excess bandwidth. —Therefore,  $-1/T_s = \omega_s/2\pi T = 1/MT - NT$  which can be solved to give  $\omega_s = 2\pi/M$  for the radian frequency sampling rate of the filter bank which is identical to the Wavelet repetition rate.

Equations (8) step 1 —QMF LS error metric  $\mu(3)$  expresses the requirement on the deadband that the PSD's from the contiguous channels in FIG. 4 —add to unity across the deadband 31 — $[\omega_p, \omega_s]$  —in FIG. 4 in order that the Wavelets be QMF filters.

Equations (8) step 1 Inter-symbol interference ISI and ACI error metrics  $\mu(4), \mu(5)$  are orthonormality metrics that measure how close we are able to designing the set of Wavelets to be orthonormal over the t-f space, with the closeness given by the

5 ISI error metric  $\mu(4)$  and the ACI error metric  $\mu(5)$ . -ISI is the non-orthogonality error between Wavelets within the same channel separated by multiples of the sampling interval  $-1/MT$  seconds where T is the sample time and M is the interval of contiguous samples. Adjacent channel interference ACI is the non-

10 orthogonality error between between Wavelets within a channel and the Wavelets in adjacent Wavelet channels at the same sample time and at sample times separated by multiples of the sample interval. -As observed as noise contributions within each sample in a given channel, -the ISI is the noise contribution due to the

15 other received Wavelets at the different timing offsets corresponding to multiples of the sampling interval. -Likewise, the ACI is the noise contribution due to the other Wavelets in adjacent Wavelet channels at the same sampling time and at multiples of the sampling interval.

20

Equations (8) step 1 ISI and ACI errors are fundamentally caused by different mechanisms and therefore have separate metrics and weights to specify their relative importance to the overall sum of the LS metrics. -ISI is a measure of the non-

25 orthogonality between the stream of Wavelets within a channel as per the construction in FIG. 4. -On the other hand, -ACI is a measure of the non-orthogonality between the Wavelets within a channel and the other Wavelets in adjacent channels. -This means the stopband -performance metric has a significant impact on the

30 ACI due to the sharp rolloff in frequency of the adjacent channel, -and the ACI metric is then a measure of the residual non-orthogonality due to the inability of the stopband rolloff in frequency from completely eliminating the ACI errors.

Equations (8) step 2 defines the weights of the LS error metrics when summed to yield the cost function J. —These weights are real and normalized to sum to unity. —They have proven to be helpful in the Wavelet design to emphasize the relative

5 importance of the individual error metric contributions to J.

Equations (8) step 3 defines the cost function J as the weighted sum of the LS error metrics and which is minimized with respect to the DFT frequency design harmonics  $\{\psi(k')\}$  to select  
10 the best LS choice for the  $\{\psi(k')\}$  to design the mother Wavelet in —3—in equations (7) and the channel Wavelets by frequency translation in —5—in equations (6)—).

Multi-scale mother Wavelet frequency response in FIG. 5 is  
15 evaluated by implementing the LS design algorithms in reference [2] that minimize the J in —3—in equations (8) to find the best set of DFT frequency design coordinates  $\{\psi(k')\}$  which give the mother Wavelet 3 in equations (7). FIG. 5 plots the PSD frequency response for the multi-scale mother Wavelet and the  
20 square-root (sq-rt) raised-cosine (r-c) waveforms with excess bandwidth  $\alpha = 0.22, 0.40$  which waveforms are extensively used for the third-generation 3G-CDMA communications as well as for other communications. Plotted are the measured PSD in dB —33—42 versus the frequency offset 34—43 from dc expressed in units of the symbol rate. —Plotted against the normalized frequency  
25 offset are the mother PSD for the new Wavelet waveform 3544, the sq-rt r-c with  $\alpha = 0.22$  3645, and —the sq-rt r-c with  $\alpha = 0.40$  3746, —. passband template for the dc filter and the stopband template for the contiguous filter, and the Nyquist rate equal to  
30 the multi-resolution complex Wavelet rate = 0.5 in units of the symbol rate. —It is believed that the multi-resolution complex Wavelets can be designed as a filter with better performance parameters than possible with any other known algorithm.

OWDMA encoding for the transmitter is defined in equations (9). Step 1 lists parameters and definitions and step 2 defines the transmitted OWDMA –encoded baseband signal  $z(i)$  for contiguous data blocks and where the symbol offsets  $\Delta$  account for symbol overlaps over the symbol  $q$  data block interval within each channel.

OWDMA encoding for transmitter (9)

1 Parameters and definitions

10  $h(i)$  = roofing filter impulse response in time  
for  $h(f)$  in 15 in FIG.3

$\psi(i)$  =  $\psi(i|p=0,q=0,k=0)$  mother Wavelet in 3  
in equations (7)  
= baseband or dc  $k=0$  Wavelet at  $p=0,q=0$

15  $N$  = number of OWDMA filters over the  $1/T$   
frequency band

$1/T$  = digital sample rate for OWDMA  
 $\geq$  complex Nyquist rate for roofing filter  $h(f)$

$NT$  = OWDMA Wavelet spacing

20  $1/NT$  = OWDMA Wavelet output rate  
= OWDMA Wavelet channel separation

$NBT$  = channels used for data and pilot  
where  $B$  is the frequency band in FIG. 3

$N(1-BT)$  = guard band channels for rolloff  
of the  $h(f)$

25  $d(k,q)$  = data modulation for user  $k$  for data block  $q$   
= encoded amplitude  $A(k|q)$  and phase  $\phi(k|q)$

$x(k|q)$  = transmitted symbol encoded with  $d(k)$   
=  $A(k|q) \exp(j\phi(k|q))$

30 Assume the  $h(f)$  is flat over the passband for  
both Tx and Rx and can be neglected

2 Transmitted OWDMA encoded baseband signal  $z(i)$

Time index field referenced to  $q=0$



$i = 0, +/-1, +/-2, . . .$   
 $=$  digital sampling time index  
 $i_0 =$  index over a  $\psi(i)$  spacing interval  
 $= 0, 1, 2, . . . , N-1$   
5  $\Delta =$  index over the  $\psi(i)$  range in units  
of the  $\psi(i)$  spacing interval  $N$   
 $= (L/2-1), . . . , -1, 0, +1, . . . , +(L/2-1)$

**Complex baseband signal**

10  $z(i_0|q) =$  complex baseband signal over  $i_0$   
for data block  $q$   
 $= \text{FWT}[ x(k|q+\Delta) ]$   
 $= N^{-1} \sum_{\Delta} \sum_k x(k|q+\Delta) \psi(i|q+\Delta, k)$   
 $= N^{-1} \sum_{\Delta} \sum_k x(k|q+\Delta) \psi(i_0+\Delta N) E^*(k, i_0)$   
 $z(i) = \sum_q z(i_0|q)$   
15 where FWT is the fast multi-resolution complex  
Wavelet transform

3 FWT algorithm for OWDMA encoding in the transmitter

3.a) FWT pre-calculation  $\text{FFT}^{-1}$

20  $\lambda(i_0, q+\Delta) = N^{-1} \sum_k x(k|q+\Delta) E^*(k, i_0)$   
 $= \text{FFT}^{-1}[ N^{-1} \sum_k x(k|q+\Delta) E^*(k, i_0) ]$

3.b) FWT post-sum

$z(i_0|q) = \sum_{\Delta} \psi(i_0+\Delta N) \lambda(i_0|q+\Delta)$

25 4 Computational complexity of fast algorithm

Real multiply rate  $R_M$

$$R_{MT} = 2 \log_2(N) + 2 L$$

Real add rate  $R_A$

$$R_{AT} = 3 \log_2(N) + 2 L$$

30

Step 3 is the new fast FWT algorithm in this invention  
disclosure for the transmitted OWDMA which consists of the pre-

calculation  $\text{FFT}^{-1}$  in sub-step 3.a followed by a post-sum in sub-step 3.b.

Step 4 evaluates the real multiply complexity metric  $-R_{MT}$  and real add computational complexity metrics  $R_{AT}$  in terms of multiplies/adds per digital sample for the fast algorithm in step 3. The first term in these metrics is the complexity of the  $-\text{FFT}^{-1}$  for a base 2 and the second term is the complexity of extending the multi-resolution complex Wavelet waveform over  $L$  -of the symbol intervals.

OWDMA decoding for the receiver is defined in equations (10). -Step 1 refers to 1,2 in equations (9) for the parameters and definitions and defines the OWDMA filtering Wavelet. Step 2 demonstrates Wavelet orthogonality. Estimates of the transmitted symbols in step 3 are equal to the  $\text{FWT}^{-1}$  of the received baseband signal.

OWDMA decoding for receiver (10)

1 Parameters and definitions are defined in 1,2 in equations (9) together with

OWDMA filtering wavelet from 5 in equations (6)

$$\begin{aligned}\psi^*(i|q,k) &= \psi^*(i|p=0,q,k) \\ &= \psi(i-qN)E(k,i_0)\end{aligned}$$

25 Assume the  $h(f)$  is flat over the passband for both Tx and Rx and can be neglected

2 Multi-resolution complex Wavelet orthogonality from 4 in equations (6)

30  **$k, k'$  orthogonality**

$$\begin{aligned}\sum_i \psi(i|q,k) \psi^*(i|q,k') &= \sum_i \psi^2(i-qN)E^*(k,i)E(k',i) \\ &= \sum_{i_0} \sum_{\Delta} [\sum_{\Delta q} \psi^2(i_0+\Delta)N] E^*(k,i_0)E(k',i_0) \\ &\cong \sum_{i_0} [1] E^*(k,i_0) E(k',i_0)\end{aligned}$$

$$= N \delta(k-k')$$

using the completeness property of the  
multi-resolution complex Wavelet from reference [2]

$$[\sum_{\Delta} \psi^2(i_0 + \Delta N)] \cong 1 \text{ for all } i_0$$

5  $q, k$  and  $q', k'$  orthogonality proven in reference  
[2] and restated in 4 in equations (6)

3 OWDMA decoding derives estimates  $\hat{x}(k|q)$  of  $x(k|q)$  for  
data block  $q$  from the receiver estimates  $\hat{z}(i)$  of  $z(i)$

$$\begin{aligned} 10 \quad \hat{x}(k|q) &= \text{FWT}^{-1}[\hat{z}(i)] \\ &= \sum_i \hat{z}(i) \psi^*(i|q, k) \end{aligned}$$

4 FWT algorithm for OWDMA decoding in receiver

4.a) FWT pre-sum

$$\begin{aligned} 15 \quad \lambda(i_0|q) &= \sum_{\Delta} \hat{z}(i_0|q+\Delta) \psi(i_0+\Delta N) \\ &= \sum_{\Delta} \hat{z}(i_0+\Delta N|q) \psi(i_0+\Delta N) \end{aligned}$$

4.b) FWT pre-sum FFT

$$\hat{x}(k|q) = \sum_{i_0} \lambda(i_0|q) E(k, i_0)$$

20 5 Computational complexity of fast algorithm

Real multiply rate  $R_M$

$$R_{MT} = 2 \log_2(N) + 2 L$$

Real add rate  $R_A$

$$R_{AT} = 3 \log_2(N) + 2 L$$

25

Step 4 is the new fast algorithm for the received OWDMA  
which partitions the baseband symbol detection  $\hat{x}(k|q)$  in step 3  
into a pre-sum calculation sub-step 4.a of  $\lambda(i_0|q)$  followed by a  
sub-step 4.b FFT of this pre-sum.

30

Step 5 evaluates the real multiply complexity metric  $-R_{MT}$   
and real add computational complexity metrics  $R_{AT}$  in terms of  
multiplies/adds per digital sample for the fast algorithm in step

3. The first term in these metrics is the complexity of the  $-FFT^{-1}$  for a base 2 and the second term is the complexity of extending the multi-resolution complex Wavelet waveform over L of the symbol intervals.

5

MS-CDMA parameters and codes are defined in equations (11). Step 1 defines the scenario parameters. Step 2 partitions the user index u field into the sub-fields  $u_0, u_1$  of size  $N_0, N_1$  for scales 0,1 respectively and which are the indices over the users within each channel and the indices over the channels within the MS-CDMA group and which uniquely represent u as  $u = u_0 + u_1 N_1$ .

10

Step 3 partitions the code chip index n field into the sub-fields  $n_0, n_1$  of size  $N_0, N_1$  for scales 0,1 respectively and which are the indices over the chips within each channel and the indices over the channels of the MS-CDMA group and which uniquely represent n as  $n = n_0 + n_1 N_1$ .

15

Step 4 defines the  $N_c \times N_c$  MS-CDMA code matrix C whose elements are  $C(u, n)$  where u+1 is the row index and n+1 is the column index and where the +1 has been added to correspond to the row and column numbering starting with +1. MS-CDMA code vector

20

MS-CDMA parameters and codes (11)

25

1 Scenario parameters

M = number of communications channels  
of band B

$N_0$  = number of CDMA chips per channel

$N_1$  = number of channels in MS-CDMA group

30

$N_c$  = number of chips in MS-CDMA group =  $N_0 N_1$

$1/T_0$  = MS-CDMA chip/symbol rate

=  $1/NT$  for OFDMA, OWDMA in FIG. 1,2

$x(u, q)$  = User u in group q

=  $x(k|q)$  when  $N_0=1$ ,  $u=k$

$$= x(k) \quad \text{when } N_0=1, u=k, q=0$$

2 User index  $u$  for a MS-CDMA group

$u$  = Index of CDMA users

$$= 0, 1, \dots, (N_c-1)$$

$= (u_0, u_1)$  field representation of  $u$

$$= u_0 + u_1 N_0 \quad \text{for fields } u_0, u_1$$

where the index fields are

$u_0$  = index of users in a channel in

10 the MS-CDMA group

$$= 0, 1, \dots, (N_0-1)$$

$u_1$  = index of channels in the MS-CDMA group

$$= 0, 1, \dots, N_1-1$$

15 3 Code chip index  $n$  for a MS-CDMA group

$n$  = Index of CDMA code chips

$$= 0, 1, \dots, (N_c-1)$$

$= (n_0, n_1)$  field representation of  $n$

$$= n_0 + n_1 N_0 \quad \text{for fields } n_0, n_1$$

20 where the index fields are

$n_0$  = index of code chips within a channel

in the MS-CDMA group

$$= 0, 1, \dots, (N_0-1)$$

$n_1$  = index of channels in the MS-CDMA group

$$25 \quad = 0, 1, \dots, (N_1-1)$$

4 MS-CDMA code matrix  $C$  for a MS-CDMA group

$C$  =  $N_c \times N_c$  MS-CDMA code matrix

$= [C(u, n)]$  matrix of elements  $\{C(u, n)\}$

30  $= [c(u)]$  matrix of  $1 \times N_c$  code vectors

$C(u)$  = MS-CDMA code vector  $u$  which is row  $u+1$  in

$C$  counting rows, columns starting with 1

$$= [C(u, 0), C(u, 1), \dots, C(u, N_c-1)]$$

35 5 ~~Kronecker product example for  $C$  for a MS-CDMA group code~~

## matrix C construction

### 5.a Non-factorable C: algebraic field construction

$$C = [ C(u, n) ]$$

5 
$$= [ C(u_0 + u_1 N_0, n_0 + n_1 N_0) ]$$

### 5.b Factorable C: Tensor product (Kronecker) construction

$$C = C_1 \otimes C_0$$

10 
$$= [ C_1(u_1, n_1) C_0(u_0, n_0) ]$$

~~$$C = C_1 \otimes C_0 \text{ Kronecker product of } C_1 \text{ and } C_0$$~~

~~$$= [ C(u, n) = C_1(u_1, n_1) C_0(u_0, n_0) ]$$~~

where

$$\otimes = \text{Kronecker (tensor) product of } C_1 \text{ and } C_0$$

15 
$$C_1 = N_1 \times N_1 \text{ MS-CDMA scale 1 code matrix}$$
  
$$= [ C_1(u_1, n_1) ] \text{ matrix of elements } \{ C_1(u_1, n_1) \}$$
  
over the user channels within the group

$$C_0 = N_0 \times N_0 \text{ MS-CDMA scale 0 code matrix}$$
  
$$= [ C_0(u_0, n_0) ] \text{ matrix of elements } \{ C_0(u_0, n_0) \}$$
  
20 over the user chips within each channel  
in the group

25  $c(u)$  for user  $u$  is the row  $u+1$  of  $C$ . -The  $C$  is a complex  
orthogonal code matrix.

Step 5 illustrates the construction of MS-CDMA code matrix  
 $C$  for non-factorable and factorable  $C$ . In 5.a for a non-  
30 factorable  $C$  the algebraic fields for code indices  $u$  and chip  
indices  $n$  is constructed as the sum of the scale "0" algebraic  
fields of the code indices  $u_0$  and chip indices  $n_0$  of users within  
each channel in the MS-CDMA group, plus the scaled addition of  
the scale "1" algebraic fields of the code indices  $u_1$  and chip

indices  $n_1$  of users over the channels of the MS-CDMA group  
wherein the scale factor is the size  $N_0$  of the scale "0" algebraic  
fields of indices. For the special case where C is factorable in  
5.b the C is constructed as ~~is an example of a MS-CDMA code~~

5 ~~matrix C which is factorable into a~~ Kronecker or Tensor product  
 $C = C_1 \otimes C_0$  ~~of  $C_1$  and  $C_0$  where " $\otimes$ " is the Kronecker or tensor~~  
product and the matrix  $C_1$  is the  $N_1 \times N_1$  MS-CDMA scale "1" code  
matrix over the user channels within the MS-CDMA group ~~and the~~  
matrix  $C_0$  is the  $N_0 \times N_0$  MS-CDMA scale "0" code matrix for the  
10 user chips within each channel in the MS-CDMA group.

~~A general definition for MS-CDMA code matrix C which~~  
~~includes the definitions in references [3],[4] is allowed by this~~  
~~invention disclosure. The example Kronecker product construction~~  
15 ~~of C is important because it is a property possessed by the real~~  
~~Walsh or Hadamard orthogonal matrices as well as the majority of~~  
~~orthogonal matrices used or proposed for CDMA and because of the~~  
~~relative ease in finding a fast algorithm for the transmitter and~~  
~~receiver coding and decoding and because of the ease in~~  
20 ~~constructing multiple scales for MS-CDMA as demonstrated in 5 in~~  
~~equations (11).~~

MS-CDMA representative application to OFDMA in FIG. 1 uses  
the candidate architecture which spreads the CDMA over the M  
contiguous 48 data channels or 52 contiguous data plus pilot  
25 channels ~~and the representative application to OFDMA in FIG. 3~~  
uses the candidate architecture which spreads the CDMA over the M  
contiguous OFDMA ~~communications~~ channels across B.

MS-CDMA partitions the M channels into channel groups of  
30 size  $N_1$  and provides a scale over the chips within the channels  
and another scale over the channels within this group. Code chip  
length  $N_c$  for each user in a channel group is equal to  $N_c = N_0 N_1$   
where  $N_0$  is the number of chips in each channel assigned to scale  
"0" and  $N_1$  is the number of channels assigned to scale "1"

within the group. -Each user has a chip scale "0" and a channel scale "1"-. Chip scale "0" spreads the data over the chips within each channel and channel scale "1" then spreads the channel chips uniformly over the  $N_1$  channels with the result that each user occupies each of the channels within the  $N_1$  channel group.

There could be from 1 to  $M/N_1$  channel groups depending on the architecture and applications. The use of multiples groups  $M/N_1 > 1$  tends to be desirable since the storage requirements and computational complexity are reduced as the number of groups are increased and the spreading advantages within each group tend to saturate as the number  $N_1 > 16$  when the channels within each group are spread across the fullband  $M$  channels.

For  $N_0=1$  there is no CDMA within each channel and the MS-CDMA then spreads the signals over each channel within a group for both OFDMA and OWDMA to function as a means to spread each channel over the fullband  $M$  channels and which may be a desirable architecture when the channels are sufficiently narrow to produce a sufficiently long pulse to counter multipath.

MS-CDMA OFDMA transmitter equations are defined in equations (12) for MS-CDMA. Step 1 gives the parameters and definitions. Step 2 defines the encoding equations for chip  $n_0$  for data block  $q$ . Sub-step 2.a uses the fast code transform FFT developed in references [3],[4] to generate the encoded vector. Sub-step 2.b uses the inverse  $FFT^{-1}$  to construct the transmitter complex baseband signal  $z(i_0|n_0+qN_0)$  for chip  $n_0$  for data block  $q$  and these signals are combined -in sub-step 2.c to generate the transmitter signal  $z(i)$  for all  $n_0, q$ .

Step 3 evaluates the real multiply complexity metric  $-R_M T$  and real add computational complexity metrics  $R_A T$  in terms of multiplies/adds per digital sample for the fast algorithm in step 3. The first term in these metrics is the complexity of the  $-FFT^{-1}$



MS-CDMA OFDMA transmitter equations (12)

1 Parameters and definitions are defined in 1 in  
equations ( 2), (9), (11) together with

5 MS-CDMA group index field  $\{g\}$

$g = 0, 1, \dots, (M/N_1-1)$  where  $M/N_1 =$  number  
of MS-CDMA groups over the M data channels

Frequency harmonic k

$$k = k(n_1|g, q)$$

10 MS-CDMA chip index fields

$$n = n_0 + n_1 N_0$$

$$= 0, 1, 2, \dots, N_c-1$$

$n_0 = 0, 1, 2, \dots, N_0-1$  scale 0 chips in channel

$n_1 = 0, 1, 2, \dots, N_1-1$  scale 1 chips over channels

15  $N_c = N_0 N_1$  number of chips in a MS-CDMA group

MS-CDMA channel code index  $n_1$

$n_1 = n_1(k|g, q)$  function of k for a given g, q

User symbol x

$$x = x(u|g, q)$$

20  $= A(u|g, q) \exp j\phi(u|g, q)$

MS-CDMS code orthogonality

$$CC^* = N_c I$$

PN spreading code  $P(n)$  for chip n in group g

$$P(n|g, q) = \exp( j\phi(n|g, q)$$

25 Assume the band and pulse filtering can be

neglected on both Tx and Rx which is

equivalent to the assumption  $p(i) \cong 1$  for all i

2 MS-CDMA OFDMA encoding for complex baseband signal  $z(i)$

30 2.a) FCT MS-CDMA encoding for data block q

$$Z(n|g, q) = \text{FCT}[ x(u|g, q) ]$$

$$= N_c^{-1} \sum_u x(u|g, q) C(u, n) P(n|g, q)$$

where FCT is the fast code transform

2.b)  $\text{FFT}^{-1}$  OFDMA encoding for chip  $n_0$  of data block q

$$z(i_0|n_0+qN_0) = \text{FFT}^{-1} [ Z(n|g,q) ]$$

$$= N^{-1} \sum_k Z(n|g,q) E^*(k, i_0)$$

$$\text{where } n = n_0 + n_1 N_0$$

$$n_1 = n_1(k|g,q)$$

$$5 \quad Z(n|g,q) = Z(n_0 + n_1(k|g,q)N_0)$$

2.c) Complex baseband transmitter signal  $z(i)$

$$z(i) = \sum_{(o)} z(i_0|n_0+qN_0) \quad \text{for } (o) = (n_0+qN_0)N$$

### 3 Computational complexity

10 Real multiply rate  $R_M$

$$R_M T = 2 \log_2(N)$$

Real add rate  $R_A$

$$R_A T = 3 \log_2(N) + 2 \log_2(N_c) + 2$$

15 for a base 2 and the second term is the complexity of the FCT assuming the FCT does not require any multiplies.

MS-CDMA OWDMA transmitter equations are defined in equations (13) for MS-CDMA. Step 1 lists the parameters and definitions. Step 2 defines the encoding equations for chip  $n_0$  for data block  $q$ . The FCT on the symbols in sub-step 2.a yields the encoded data block  $Z(n|g,q)$  and the FWT on this output in sub-step 2.b yields the transmitter complex baseband signal  $z(i_0|n_0+qN_0)$  for chip  $n_0$  for data block  $q$  and these signals are combined in sub-step 2.c to generate the transmitter signal  $z(i)$  for all  $n_0, q$ .

MS-CDMA OWDMA transmitter equations (13)

1 Parameters and definitions are defined in 1 in  
 30 equations (9), (12) together with the range of  $q$   
 $q = 0$  for the data block being addressed  
 $= +/-1$  contiguous data blocks  
 Assume the  $h(f)$  is flat over the passband for  
 both Tx and Rx and can be neglected

2 MS-CDMA OWDMA encoding for chip  $n_0$  for data block  $q$

2.a) FCT MS-CDMA encoding for data block  $q$

$$\begin{aligned} Z(n|g,q) &= \text{FCT}[x(u|g,q)] \\ &= N_c^{-1} \sum_u x(u|g,q) C(u,n) P(n|g) \end{aligned}$$

2.b) FWT OWDMA encoding for chip  $n_0$  of data block  $q$

$$\begin{aligned} z(i_0|n_0+qN_0) &= \sum_{\delta q} \sum_{\Delta} \text{FWT}[Z(n+\Delta|g,q+\delta q)] \\ &= N^{-1} \sum_k \sum_{\delta q} \sum_{\Delta} Z(n+\Delta|g,q+\delta q) \psi(i|n_0+\Delta,k) \\ &= N^{-1} \sum_k \sum_{\delta q} \sum_{\Delta} Z(n+\Delta|g,q+\delta q) \psi(i_0+\Delta N) E^*(k,i_0) \end{aligned}$$

where the overlap conditions on  $\delta q$  are

$$\begin{aligned} \delta q &= \Delta \quad \text{for } N_0=1 \\ &= \text{sign}(\Delta+n_0) \lfloor |\Delta+n_0|/N_0 \rfloor \quad \text{otherwise} \\ \text{sign}(o) &= \text{numerical sign of } (o) \\ \lfloor (o) \rfloor &= \text{rounded down integer value of } (o) \end{aligned}$$

where the overlap index  $\Delta = 0, +/-1, .$

. . . , +/- (L/2-1)

2.c) Complex baseband transmitter signal  $z(i)$

$$z(i) = \sum_{(o)} z(i_0|n_0+qN_0) \quad \text{for } (o)=(n_0+qN_0)N$$

3 FWT algorithm for OWDMA encoding in the transmitter

3.a) FWT pre-calculation  $\text{FFT}^{-1}$

$$\begin{aligned} \lambda(i_0|n_0+\Delta,q) &= N^{-1} \sum_k Z(n+\Delta|g,q) E^*(k,i_0) \\ &= \text{FFT}^{-1}[N^{-1} \sum_k Z(n+\Delta|g,q) E^*(k,i_0)] \end{aligned}$$

3.b) FWT post-sum

$$z(i_0|n_0) = \sum_q \sum_{\Delta} \psi(i-(n_0+\Delta)N) \lambda(i_0|n_0+\Delta,q)$$

4 Computational complexity for fast algorithms in 2.a,3

Real multiply rate  $R_M$

$$R_M T = 2 \log_2(N) + 2 L$$

Real add rate  $R_A$

$$R_A T = 3 \log_2(N) + 2 L + 2 \log_2(N_c) + 2$$

Step 3 is the new fast FWT algorithm in this invention disclosure for the transmitted OWDMA which consists of the pre-calculation  $FFT^{-1}$  in sub-step 3.a followed a post-sum in sub-step 3.b of the product from sub-step 3.a with the corresponding Wavelet overlaps over the  $q$  data block interval.

Step 4 evaluates the real multiply complexity metric  $-R_{MT}$  and real add computational complexity metrics  $R_{AT}$  in terms of multiplies/adds per digital sample for the fast algorithms FCT and FWT in steps 2,3. The first term in these metrics is the complexity of the  $-FFT^{-1}$  for a base 2, the second term  $2L$  is the complexity of extending the multi-resolution complex Wavelet waveform over  $L$  of the MT symbol intervals, and the remaining terms are the complexity of the FCT assuming the FCT does not require any multiplies.

MS-CDMA OFDMA/OWDMA encoding for the transmitter in FIG. 6 is a representative implementation of the MS-CDMA OFDMA encoding algorithms in equations (12) and the MS-CDMA OWDMA encoding algorithms in equations (13). Signal processing starts with the input stream of data encoded symbols  $x(u|g,q)$  from the transmitter symbol encoder 52 in FIG. 7A and defined in 1 in equations (12) for both OFDMA and OWDMA.

25

FIG. 6 MS-CDMA encoding 41 implements the fast ~~epde-code~~ transform FCT encoding defined in sub-step 2.a in equations (12), (13) for the MS-CDMA encoding and PN cover or spreading encoding to generate the  $N_c$  outputs  $x(u|g,q)C(u,n)P(n|g)$  for each MS-CDMA group  $g$  and data block  $q$ . For each group these outputs are summed 42 over  $u$  to generate the encoded vector  $Z(n|g,q)$  in 43.

OFDMA processing 44 performs an  $FFT^{-1}$  on the received set of vectors  $Z(n|g,q)$  and a summation to implement sub-steps 2.b, 2.c

35

in equations (12) and the output is band filtered 46 to generate the MS-CDMA OWDMA encoded complex baseband signal  $z(i)$  in 47.

OWDMA processing 45 performs an FWT on the received set of  
5 vectors  $Z(n|g,q)$  and a summation to implement sub-steps 2.b, 2.c in equations (13) and the output is band filtered 46 to generate the MS-CDMA OWDMA encoded complex baseband signal  $z(i)$  in 47.

Outputs  $z(i_0|n_0)$  47 from the MS-CDMA OFDMA and MS-CDMA OWDMA  
10 are digital-to-analog DAC converted 48 and handed off to the analog front end 49 as the complex baseband analog signal  $z(t)$  in 49.

MS-CDMA OFDMA/OWDMA transmitter description in FIG. 7  
15 presents a block diagram in FIG. 7A and a representative MS-CDMA mapping in FIG. 7B. FIG. 7A is a representative transmitter implementation of the MS-CDMA OFDMA and MS-CDMA OWDMA encoding in  
the shaded box in FIG. 6. The transmitter block diagram in FIG. 7A includes the FIG. 6 MS-CDMA OFDMA/OWDMA encoding in an  
20 abbreviated format ~~in the shaded box~~ 54 in FIG. 7A. FIG. 7A signal processing starts with the stream of user input data words. -Frame processor -51 -accepts these data words and performs the encoding and frame formatting wherein CRC is a  
cyclic redundant code for error detection, -and passes the  
25 outputs to the symbol encoder -52 -which encodes the frame symbols into amplitude (Ampl.) and phase coded symbols  $x(u|g,q)$  53 which are the input to the MS-CDMA encoding 55 and which is 41, 42 in FIG. 6.

30 MS-CDMA FCT encoding outputs  $Z(n|g,q)$  56 are handed over to the OFDMA and OWDMA processing 57 which performs an inverse  $FFT^{-1}$  followed by a band filtering for OFDMA which is 44, 46 in FIG. 4 and performs an inverse  $FWT^{-1}$  followed by a band filtering which is 45, 46 in FIG. 6. -This complex baseband signal  $z(i)$  in -47 in  
35 FIG. 6 is -digital-to-analog DAC converted -59 and the ~~the~~ output

complex baseband analog signal  $z(t)$  60 is handed off to the analog front end 61.

5 The  $z(t)$  is single sideband upconverted, amplified, and transmitted (Tx) —by the analog front end 61 as the real waveform — $v(t)$  —62 —at the carrier frequency  $f_0$  whose amplitude is the real part of the complex envelope of the baseband waveform  $z(t)$  and the phase angle  $\phi$  accounts for the phase change from the baseband signal to the transmitted signal. —Output waveform —62  
10 from the analog front end is the Tx waveform from the Tx antenna.

FIG. 7B illustrates a representative MS-CDMA uniform mapping of each data symbol over frequency, time, antennas, and beams of a cellular communications transmitter. Multiple  
15 antennas and beams are used when a multiple-input-multiple-output MIMO communications link is being implemented. The algebraic field construction of the algebraic index fields for the codes and chips for a 2-scale MS-CDMA construction of a non-factorable code matrix C in equations (11) represented by 151 and 152 in  
20 FIG. 7B for the algebraic chip indices  $n_0, n_1$  is continued in FIG. 7B to include the algebraic chip index fields for chips  $n_2$  in 153 over the frequency bands, chips  $n_3$  in 154 over the data blocks, chips  $n_4$  in 155 over the transmit antenna beams, and chips  $n_5$  in 156 over the transmit antennas. The corresponding algebraic code  
25 indices are respectively  $u_0, u_1, u_2, u_3, u_4, u_5$  and the MS-CDMA code length is  $N_c = N_0 N_1 N_2 N_3 N_4 N_5$  chips.

It should be obvious to anyone skilled in the communications art that this —example implementation—in FIG.  
30 6,7 clearly —defines the fundamental MS-CDMA OFDMA and MS-CDMA OWDMA —signal processing relevant to this invention disclosure and —it is obvious that this example is representative of the other possible signal processing approaches.

MS-CDMA OFDMA receiver equations are defined in equations (14). –Step 1 lists the parameters and definitions and the assumption that the band and pulse filtering can be neglected. Step 2 –defines decoding of the received chip signal to derive the estimate  $\hat{x}(u|g,q)$  of the transmitted symbol  $x(u|g,q)$ . Sub-step 2.a derives the estimate for the encoded symbols using the FFT and sub-step 2.b uses the  $FCT^{-1}$  –on –this estimate to derive the transmitted symbol estimate.

10 MS-CDMA OFDMA receiver equations (14)

1 Parameters and definitions are defined in 1 in equations (2),(9),(11) together with

$\hat{z}(i)$  = receiver band filter estimate of the transmitted complex baseband signal  $z(i)$

15 Assume the band and pulse filtering can be neglected on both Tx and Rx which is equivalent to the assumption  $p(i) \cong 1$  for all  $i$

2 MS-CDMA OFDMA decoding to derive data symbol  $\hat{x}(u|g,q)$

20 2.a) FFT of each received chip vector  $\hat{z}(i_0|n_0+qN_0)$

$\hat{Z}(n|g,q)$  = estimate of transmitted  $Z(n|g,q)$   
 $= FFT[ \hat{z}(i_0|n_0+qN_0) ]$   
 $= \sum_{i_0} \hat{z}(i_0|n_0+qN_0) E(k, i_0)$

where  $\hat{z}(i_0|n_0+qN_0) = \hat{z}(i=i_0+(n_0+qN_0)N)$

25 2.b)  $FCT^{-1}$  of each group  $g$  encoded vector  $Z(n|g,q)$

$\hat{x}(u|g,q) = FCT^{-1}[ \hat{Z}(n|g,q=0) ]$   
 $= \sum_n \hat{Z}(n|g,q) P^*(n|g) C^*(u,n)$

3 Computational complexity is the same as calculated in 3 in equations (12) for the transmitter

MS-CDMA OFDMA receiver equations are defined in equations (15). Step 1 lists the parameters and definitions and –the assumption that the band and pulse filtering can be neglected.

MS-CDMA OWDMA receiver equations (15)

1 Parameters and definitions are defined in 1 in  
equations (2),(9),(11),(14) together with

5  $\hat{z}(i)$  = receiver band filter estimate of  
the transmitted complex baseband signal  $z(i)$   
Assume the  $h(f)$  is flat over the passband for  
both Tx and Rx and can be neglected

10 2 MS-CDMA OWDMA decoding to derive data symbol  $\hat{x}(u|g,q)$

2.a) FWT<sup>-1</sup> of each received signal  $\hat{z}(i)$

$$\begin{aligned}\hat{Z}(n|g,q) &= \text{estimate of transmitted } Z(n|g,q) \\ &= \text{FWT}^{-1}[\hat{z}(i)] \\ &= \sum_i \hat{z}(i) \psi^*(i|q,k)\end{aligned}$$

15 
$$= \sum_{i_0} \sum_{\Delta} \hat{z}(i_0|n_0+(q+\Delta)N) \psi(i_0+\Delta N) E(k, i_0)$$

where  $\hat{z}(i_0|n_0+qN_0) = \hat{z}(i=i_0+(n_0+qN_0)N)$

$k \rightarrow g$  which means  $k$  specifies  $g$

$n_0, k \rightarrow n=n_0+n_1(k|g,q)$

20 2.b) FCT<sup>-1</sup> of each group  $g$  encoded vector  $Z(n|g,q)$

$$\begin{aligned}\hat{x}(u|g,q) &= \text{FCT}^{-1}[\hat{Z}(n|g,q)] \\ &= \sum_n \hat{Z}(n|g,q) P^*(n|g) C^*(u,n)\end{aligned}$$

3 FWT<sup>-1</sup> algorithm

25 2.a) FWT<sup>-1</sup> pre-sum

$$\lambda(i_0|g,q) = \sum_{\Delta} \hat{z}(i_0|n_0+(q+\Delta)N) \psi(i_0-(n_0+\Delta)N)$$

where  $n_0+\Delta \rightarrow g$  using the boundary conditions  
in 2.b in equations (13)

2.b) FWT<sup>-1</sup> pre-sum FFT

30 
$$\begin{aligned}\hat{Z}(n|g,q) &= \sum_{i_0} \text{FFT}[\lambda(i_0|g,q)] \\ &= \sum_{i_0} \lambda(i_0|g,q) E(k, i_0)\end{aligned}$$

3 Computational complexity is the same as calculated



in 3 in equations (13) for the transmitter

Step 2 defines decoding of the received chip signal to derive the estimate  $\hat{x}(u|g,q)$  of the transmitted symbol  $x(u|g,q)$ . Sub-  
5 step 2.a derives the estimate for the encoded symbols using the  $\text{FWT}^{-1}$  and sub-step 2.b uses the  $\text{FCT}^{-1}$  on this estimate to derive the transmitted symbol estimate. —Step 3 is the new fast FWT algorithm in this invention disclosure for the received MS-CDMA OWDMA which consists of the pre-sum in sub-step 3.a followed an  
10 FFT on this pre-sum.

MS-CDMA OFDMA/OWDMA decoding for the receiver in FIG. 8 is a representative implementation of the MS-CDMA OFDMA decoding algorithms in equations (14) and the MS-CDMA OWDMA decoding  
15 algorithms in equations (15). Signal processing starts with the input intermediate frequency IF signal after being single sideband downconverted and synchronized in frequency —63 from the receiver front end —72 in FIG. 9. —This input signal is band filtered 64 and handed off to the analog-to-digital converter ADC  
20 or A/D 65 whose digital output is the received estimate  $\hat{z}(i)$  66 of the transmitted complex baseband signal  $z(i)$  60 in FIG. 7. For OFDMA —67 this received signal is processed by an FFT to derive estimates  $\hat{Z}(n|g,q)$  68 of the MS-CDMA encoded signal  $Z(n|g,q)$  56 in FIG. 7. For OWDMA 67 —this received signal is  
25 processed by an inverse multi-resolution complex Wavelet transform  $\text{FWT}^{-1}$  to derive estimates  $\hat{Z}(n|g,q)$  68 of the MS-CDMA encoded signal  $Z(n|g,q)$  56 in FIG. 7 implementing the fast algorithm defined in 2 in equations (15). —Recovered estimates  $\hat{Z}(n|g,q)$  68 are processed by the inverse fast code transform  
30  $\text{FCT}^{-1}$  69 to derive estimates  $\hat{x}(u|g,q)$  70 of the transmitted data symbols  $x(u|g,q)$  53 in FIG. 7 for hand over to the symbol decoder.

MS-CDMA OFDMA/OWDMA receiver block diagram in FIG. 9 is a representative receiver implementation of the MS-CDMA OFDMA and MS-CDMA OWDMA decoding ~~in the shaded box in FIG. 8~~. The receiver block diagram in FIG. 9 includes the FIG. 8 MS-CDMA OFDMA/OWDMA decoding in an abbreviated format ~~in the shaded box 74~~ in FIG. 9. FIG. 9 signal processing starts with the received Rx waveform 71 from the transmitter 62 in FIG. 7. Received (Rx) signal  $\hat{v}(t)$  71 is an estimate of the transmitted signal  $v(t)$  62 in FIG. 7 received with errors in time  $\Delta t$ , frequency  $\Delta f$ , phase  $\Delta \theta$ , and with an estimate  $\hat{z}(i)$  of the transmitted complex baseband signal  $z(t)$  60 in FIG. 7. This received signal  $\hat{v}(t)$  is amplified and downconverted by the analog front end 72 and single side band SSB downconverted and synchronized 73 and handed over to the digital-to-analog conversion DAC processing 75 for band filtering by  $h(f)$  and digitization to generate the baseband signal  $\hat{z}(i)$  76 which is the received estimate of the transmitted signal  $z(i)$  60 in FIG. 7. Timing synchronization could be implemented in the DAC.

Outputs  $\hat{z}(i)$  are processed by the MS-CDMA OFDMA/OWDMA decoding to derive estimates  $\hat{x}(u|g,q)$  79 of the transmitted symbols  $x(u|g,q)$  53 in FIG. 7 and part of the information is handed off to the synchronization (sync) processor. For the inverse MS-CDMA<sup>-1</sup> OFDMA<sup>-1</sup> the processing 79 consists of the FFT to recover estimates  $\hat{Z}(n|g,q)$  of  $Z(n|g,q)$  followed by an inverse FCT<sup>-1</sup> to recover  $\hat{x}(u|g,q)$ . For the inverse MS-CDMA<sup>-1</sup> OWDMA<sup>-1</sup> the processing 79 consists of the inverse FWT<sup>-1</sup> to recover estimates  $\hat{Z}(n|g,q)$  of  $Z(n|g,q)$  followed by an inverse FCT<sup>-1</sup> to recover  $\hat{x}(u|g,q)$  and the ~~FWT<sup>-1</sup> implements the fast transform in 2 in~~ equations (15). Outputs  $\hat{x}(u|g,q)$  are processed by the symbol decoder 80 and the frame processor 81 for handoff as the received Rx data 82.

It should be obvious to anyone skilled in the communications art that this —example implementation—in FIG. 8,9 clearly —defines the fundamental MS-CDMA OFDMA and MS-CDMA OWDMA —signal processing relevant to this invention disclosure and it is obvious that this example is representative of the other possible signal processing approaches.

Variable power control across the frequency band can be implemented by assigning each group  $g$  of transmit Tx signals their own power level  $P(g)$ . Each MS-CDMA group  $g$  occupies a subset of the channels over the frequency band  $B$  consisting of  $N_1$  channels which means that the users within group  $g$  are transmitted with the same Tx power. —On receive each group  $g$  of channels is processed separately so there is no cross-talk between the users in the different groups. The OWDMA was designed to support large dynamic range imbalances between channels which could be present with power level control. —MS-CDMA OFDMA/OWDMA variable power control over the —frequency subbands corresponding to the MS-CDMA groups supports the potential for diversity improvements by allocation of the available power to emphasize the 'best' set of available subbands, —which subbands are not required to be contiguous as well as supporting the simultaneous support of multiple users with differing power requirements due to range, multi-path, —and path attenuation effects.

A second configuration for variable power control is described in reference [2] and increases the flexibility of power control to all of the individual channels.

Preferred embodiments in the previous description is provided to enable any person skilled in the art to make or use the present invention. —The various modifications to these embodiments will be readily apparent to those skilled in the art, and the generic principles defined herein may be applied to other embodiments without the use of the inventive faculty. —Thus, the

present invention is not intended to be limited to the embodiments shown herein but is not to be accorded the wider scope consistent with the principles and novel features disclosed herein.

5

#### REFERENCES

- ~~[1] IEEE 802.11g standard~~
- 10 ~~[2] Patent filed 09/826,118 01/09/2001 New multi-resolution waveforms, U.A. von der Embse~~
- ~~[3] Patent filed 29/143,845 01/09/2001 Multi-scale CDMA, U.A. von der Embse~~
- ~~[4] Patent filed 09/826,117 01/09/2001 Hybrid-Walsh codes~~
- 15 ~~for CDMA, U.A. von der Embse~~
- ~~[5] "Multirate Digital Signal Processing", R.E. Crochiere, L.R. Rabiner, 1983, Prentice-Hall~~
- ~~[6] "Multirate Systems and Filter Banks", R.P. Vaidyanathan, 1993, Prentice-Hall~~
- 20 ~~[7] "Wavelets and Filter Banks", Gilbert Strang, Truong Nguyen, 1996, Wellesley-Cambridge Press~~
- ~~[8] Ronald R. Coifman, Yves Meyer, Victor Wickerhauser, "Wavelet analysis and signal processing", in "Wavelets and Their Applications", Jones & Bartlett Publishers, 1992~~
- 25 ~~[9] T. Blu, "A new design algorithm for two-band orthonormal rational filter banks and orthonormal rational Wavelets", IEEE Signal Processing, June 1998, pp. 1494-1504~~
- ~~[10] M. Unser, P. Thevenaz, and A. Aldroubi, "Shift-orthogonal Wavelet bases", IEEE Signal Processing, July 1998, pp. 1827-1836~~
- 30 ~~[11] K.C. Ho and Y. T. Chan, "Optimum discrete Wavelet scaling and its application to delay and Doppler estimation", IEEE Signal Processing, Sept. 1998, pp. 2285-2290~~
- ~~[12] I. Daubechies, "Ten Lectures on Wavelets", Philadelphia: SIAM, 1992~~

present invention is not intended to be limited to the embodiments shown herein but is not to be accorded the wider scope consistent with the principles and novel features disclosed herein.

5

#### REFERENCES

- ~~{1} IEEE 802.11g standard~~
- 10 ~~{2} Patent filed 09/826,118 01/09/2001 New multi-resolution waveforms, U.A. von der Embse~~
- ~~{3} Patent filed 29/143,845 01/09/2001 Multi-scale CDMA, U.A. von der Embse~~
- ~~{4} Patent filed 09/826,117 01/09/2001 Hybrid-Walsh codes~~
- 15 ~~for CDMA, U.A. von der Embse~~
- ~~{5} "Multirate Digital Signal Processing", R.E. Crochiere, L.R. Rabiner, 1983, Prentice-Hall~~
- ~~{6} "Multirate Systems and Filter Banks", R.P. Vaidyanathan, 1993, Prentice-Hall~~
- 20 ~~{7} "Wavelets and Filter Banks", Gilbert Strang, Truong Nguyen, 1996, Wellesley-Cambridge Press~~
- ~~{8} Ronald R. Coifman, Yves Meyer, Victor Wickerhauser, "Wavelet analysis and signal processing", in "Wavelets and Their Applications", Jones & Bartlett Publishers, 1992~~
- 25 ~~{9} T. Blu, "A new design algorithm for two-band orthonormal rational filter banks and orthonormal rational Wavelets", IEEE Signal Processing, June 1998, pp. 1494-1504~~
- ~~{10} M. Unser, P. Thevenaz, and A. Aldroubi, "Shift-orthogonal Wavelet bases", IEEE Signal Processing, July 1998, pp. 1827-1836~~
- 30 ~~{11} K.C. Ho and Y. T. Chan, "Optimum discrete Wavelet scaling and its application to delay and Doppler estimation", IEEE Signal Processing, Sept. 1998, pp. 2285-2290~~
- ~~{12} I. Daubechies, "Ten Lectures on Wavelets", Philadelphia: SIAM, 1992~~

~~{13} P.P. Vaidyanathan and T.Q. Nguyen, "Eigenvalues: A New  
Approach to Least Squares FIR Filter Design and Applications  
Including Nyquist Filters", IEEE Trans. on Circuits and Systems,  
Vo. CAS-34, No. 1, Jan. 1987, pp 11-23~~

5 ~~{14} J.H. McClellan, T.W. Parks and L.R. Rabiner, "A Computer  
Program for Designing Optimum FIR Linear Phase Filters", IEEE  
Trans Audio Electroacoust. Vol. AU-21, Dec. 1973, pp. 506-526~~

10

15

#### **DRAWINGS**